Scenarios in Event Bushes - A formal approach

Uwe Wolter

University of Bergen, Norway

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My background

1981-88 TU Magdeburg (East-Germany)

- Study of mathematics in Magdeburg and Jena
- First scientific paper in Formal Language Theory as a student
- PhD in the Theory of Partial Algebras.

1988-91 Humboldt-University Berlin

- Logic and functional programming
- Visiting PostDoc, Laboratory for Foundations of Computer Science, University of Edinburgh (6 month)
  - Algebraic specifications and Categorical Algebra
  - Type theory

My background

1991-2000 TU Berlin

- Algebraic specifications
- Category theory and its application in Formal Specifications
- Abstract model theory (institutions)
- Coalgebras (and Process calculi)
- Graph transformations
- (Petri nets)

2000-today University of Bergen (Norway)

- (Meta) Modelling in Software Engineering
- (Foundations of) Diagrammatic specification formalisms based on the Generalized Sketch Framework (Zinovy Diskin, Riga, 90's)
- Diagram Predicate Framework (DPF)

  $\Rightarrow$ Cyril catched me here!

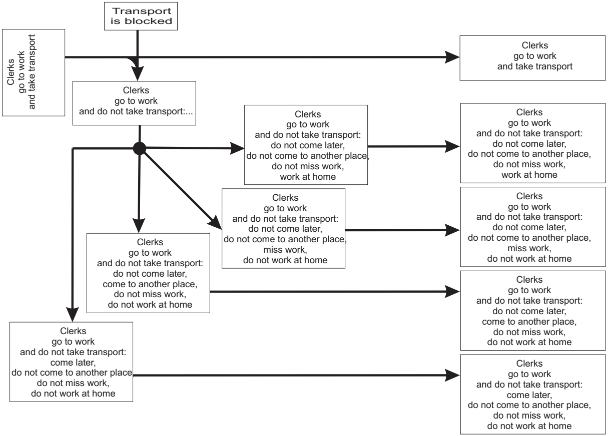
Why I'm here today?

- Cyril insisted on a meeting in Bergen in 2013 and introduced me to Event Bushes.
- I dislike that science has become a "publication industry".
- I enjoy intense scientific discussions and I like to help people that burn for an idea/topic.
- I'm interested to understand and to formalize "structures" and the relations between different kinds of "structures".
- Event bushes show structural features that I have not seen before.

My position and intended role

- I'm a mathematician with a decent background in various kinds of formal specifications on various abstraction levels.
- I'm specialized in studying and formalizing "structures".
- My role?
  - Develop an appropriate mathematical foundation of event bushes.
  - Based on this, consolidate and extend the method of event bushes.

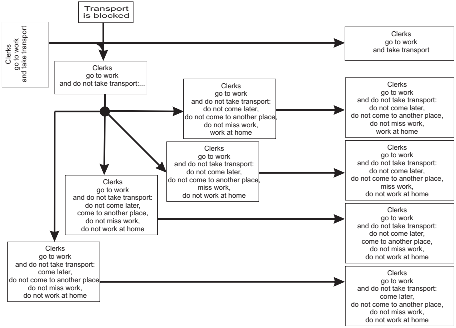
What (abstract) structures Event Bushes are based upon?



Answer 1

An event bush $\mathcal{E}$ is a (simple) **directed graph** with a set $\mathcal{E}_V$ of vertexes (nodes) and a set $\mathcal{E}_E \subseteq \mathcal{E}_V \times \mathcal{E}_V$ of edges $(x, y) \in \mathcal{E}_V \times \mathcal{E}_V$.

More structural features?



Observation 1

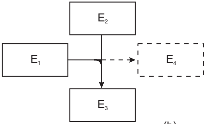
An event bush has, in principal, two types of vertexes - event vertexes and connector vertexes.

(Small) Problem

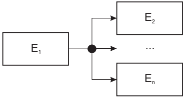
The flux connector is not visualized by a special vertex. And influx comprises actually two connector vertexes.

Connectors represented by connector vertexes

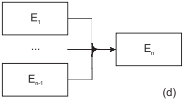
(a)



(b)



(c)



(d)

$$\Longrightarrow$$

$$E_1 \longrightarrow fl \longrightarrow E_2$$

$$\begin{tikzcd}
 & E_2 \arrow[d] & & \\
E_1 \arrow[r] \arrow[rr, bend left] & ifl \arrow[d] & fl \arrow[r] & E_4 \\
 & E_3 & &
\end{tikzcd}$$

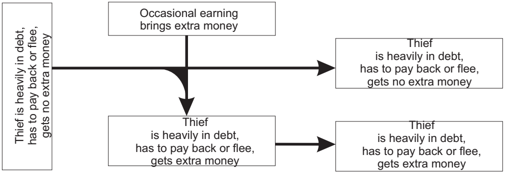
$$\begin{tikzcd}
 & & E_2 \\
E_1 \arrow[r] & frc \arrow[ur] \arrow[r] \arrow[dr] & \cdots \\
 & & E_n
\end{tikzcd}$$

$$\begin{tikzcd}
E_1 \arrow[dr] & & \\
\cdots \arrow[r] & cfl \arrow[r] & E_n \\
E_{n-1} \arrow[ur] & &
\end{tikzcd}$$

(Abstract) Structure of event bushes?

Answer 1 - revised

The **structure** of an event bush is given by a directed **graph** $\mathcal{E}$ with a set $\mathcal{E}_V$ of vertexes (nodes) and a set $\mathcal{E}_E$ of edges $(x, y) \in \mathcal{E}_V \times \mathcal{E}_V$.



$\Longrightarrow$

$$
\begin{tikzcd}
 & E_2 \arrow[d] & & \\
E_1 \arrow[r] \arrow[rr, bend left] & ifl_1 \arrow[d] & fl_1 \arrow[r] & E_4 \\
 & E_3 \arrow[r] & fl_2 \arrow[r] & E_5
\end{tikzcd}
$$

(Abstract) Structure of event bushes?

Observation 1 - revised

The structure graph $\mathcal{E} = (\mathcal{E}_V, \mathcal{E}_E)$ of an event bush is a **bipartite** graph:

- $\mathcal{E}_V$ is divided into two disjoint sets – a set $\mathcal{E}_{Event}$ of "event nodes" and a set $\mathcal{E}_{Conn}$ of "connector nodes".
- Edges go either from "event nodes" to "connector nodes" or vice versa. No edges between events and no edges between connectors!

$$
\begin{tikzcd}
 & E_2 \arrow[d] & & \\
E_1 \arrow[r] \arrow[rr, bend left] & ifl_1 \arrow[d] & fl_1 \arrow[r] & E_4 \\
 & E_3 \arrow[r] & fl_2 \arrow[r] & E_5
\end{tikzcd}
$$

Question

How can we represent the property **"bipartite"** by means of graphs?

Bipartite structure graphs

Concept "bipartite"

The concept **"bipartite"** can be represented by a graph $\mathcal{B}$ with $\mathcal{B}_V = \{Event, Conn\}$ and $\mathcal{B}_E = \{(Event, Conn), (Conn, Event)\}$:

$$\mathcal{B}: \qquad \begin{tikzcd} Event \arrow[r, bend left] & Conn \arrow[l, bend left] \end{tikzcd}$$

Structure graph as bipartite graph - Definition

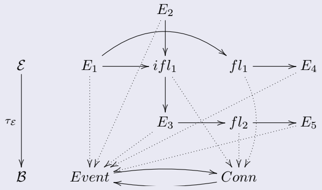
The requirement that the structure graph $\mathcal{E} = (\mathcal{E}_V, \mathcal{E}_E)$ of an event bush is **bipartite** can be expressed by requiring a **graph homomorphism** $\tau_\mathcal{E} : \mathcal{E} \to \mathcal{B}$, i.e., a function $\tau_\mathcal{E} : \mathcal{E}_V \to \mathcal{B}_V$ such that each edge $(x, y) \in \mathcal{E}_E$ in $\mathcal{E}$ entails an edge $(\tau_\mathcal{E}(x), \tau_\mathcal{E}(y)) \in \mathcal{B}_E$ in $\mathcal{B}$.

Since $\mathcal{B}_V$ has two elements $\tau_\mathcal{E}$ generates a partition of $\mathcal{E}_V$ into the two disjoint sets $\mathcal{E}_{Event} := \tau_\mathcal{E}^{-1}(Event) = \{x \in \mathcal{E}_V \mid \tau_\mathcal{E}(x) = Event\}$ and $\mathcal{E}_{Conn} := \tau_\mathcal{E}^{-1}(Conn) = \{x \in \mathcal{E}_V \mid \tau_\mathcal{E}(x) = Conn\}$.

Structure graph as bipartite graph - Example

Example

For our example the graph homomorphism $\tau_{\mathcal{E}} : \mathcal{E} \to \mathcal{B}$, that turns the structure graph $\mathcal{E}$ into a bipartite graph, is given by the assignments $\tau_{\mathcal{E}}(E_1) = \tau_{\mathcal{E}}(E_2) = \tau_{\mathcal{E}}(E_3) = \tau_{\mathcal{E}}(E_4) = \tau_{\mathcal{E}}(E_5) = \mathit{Event}$ and $\tau_{\mathcal{E}}(\mathit{ifl}_1) = \tau_{\mathcal{E}}(\mathit{fl}_1) = \tau_{\mathcal{E}}(\mathit{fl}_2) = \mathit{Conn}$.



Forbidden edges

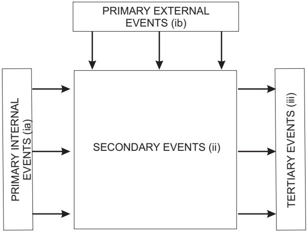
Note, that we can not map, for example, E_1 and ifl_1 to Event since there is no edge $(\tau_{\mathcal{E}}(E_1), \tau_{\mathcal{E}}(\mathit{ifl}_1)) = (\mathit{Event}, \mathit{Event})$ in $\mathcal{B}$.

Multiflow structure

A characteristic feature of event bushes is the **multiflow structure**!

Multiflow structure as graph

The informal picture can be formalized by a (bipartite) graph $\mathcal{S}$ if we interprete the "boxes" as vertexes and take into account the connectors living in the big central box.

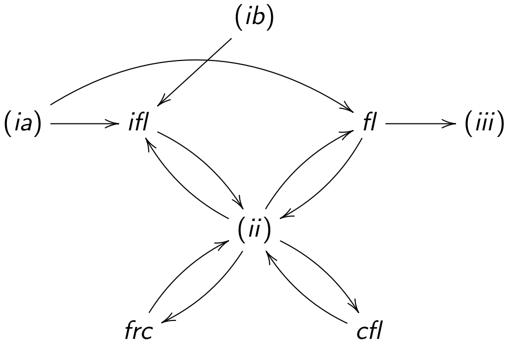


$\Longrightarrow$

$$\begin{tikzcd}
 & (ib) \arrow[d] & \\
(ia) \arrow[r] & C \arrow[r] \arrow[d, bend left] & (iii) \\
 & (ii) \arrow[u, bend left] &
\end{tikzcd}$$

Multiflow structure & connector types

If we want to declare, besides the multiflow structure, also the four different types of connectors in an event bush (together with some of Cyril's restrictions) we can use instead of $\mathcal{S}$ the more refined graph $\mathcal{ST}$ with the obvious homomorphism $\tau_{\mathcal{ST}} : \mathcal{ST} \to \mathcal{B}$.



Event bushes vs. graphs

Structure of event bushes

The structure of an event bush is (based upon) a directed graph $\mathcal{E}$ together with

- a graph homomorphism $\tau_{\mathcal{E}} : \mathcal{E} \to \mathcal{B}$, that declares the vertexes in $\mathcal{E}$ either as event vertexes or as connector vertexes, respectively,
- and a graph homomorphism $\mu_{\mathcal{E}} : \mathcal{E} \to \mathcal{ST}$, that declares as well the multiflow structure of the event bush as the types of connectors,
- where the obvious compatibility $\mu_{\mathcal{E}}; \tau_{\mathcal{ST}} = \tau_{\mathcal{E}}$ holds.

$$\begin{tikzcd} \mathcal{E} \arrow[d, "\mu_{\mathcal{E}}"'] \arrow[dr, "\tau_{\mathcal{E}}"] & \\ \mathcal{ST} \arrow[r, "\tau_{\mathcal{ST}}"'] & \mathcal{B} \end{tikzcd}$$

First summary

Event bushes in view of (simple) graphs

We have introduced two very basic formal concepts (mathematical tools) - (simple) directed graphs and homomorphisms between them.

By means of these two concepts we have been able to formalize the basic structural features of event bushes.

(Simple) Directed Graphs

A (simple) directed graph $\mathcal{G} = (G_V, G_E)$ consists of a set G_V of vertexes (or nodes) and a set $G_E \subseteq G_V \times G_V$ of edges (or arrows) where each edge is an ordered pair of vertices $(x, y) \in G_V \times G_V$.

Graph homomorphism

A graph homomorphism $\phi : \mathcal{G} \to \mathcal{H}$ is given by a function $\phi_V : G_V \to H_V$ from the vertex set of $\mathcal{G}$ to the vertex set of $\mathcal{H}$ such that each edge $(x, y) \in G_E$ entails an edge $(\phi_V(x), \phi_V(y)) \in H_E$.

Research project

Event bushes in view of (simple) graphs

For (simple) directed graphs and graph homomorphisms we do have many concepts, constructions and results available.

Let us exploit this "toolbox" to formalize and to deal with other features, relations and desired constructions in the area of event bushes.

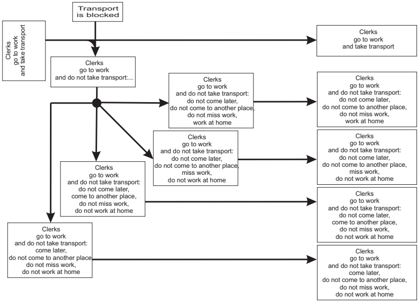
Plan for today

Let us play around a bit with graphs and graph homomorphisms to discuss the proclaimed purpose of event bushes.

Cyril A. Pshenichny:

"The event bush addresses a particular yet very wide type of geoenvironment, that of **directed alternative changes**, which is likely to occur in many information domains."

Alternatives



Question:

Where are the "alternatives" in an event bush?

Answer

A furcation connector results in "mutually incompatible events" and an influx connector is "always accompanied by a flux connector" meaning "that the initial event would not change unless affected by another event".

Alternative Scenarios

Methodological hypothesis:

An event bush describes, in a compact way, a set of alternative "scenarios" of directed changes of a (geo)environment.

Question:

How should we define a "scenario"?

Intuitive answer

Heuristics: All the behavioural aspects of an event bush are reflected in the structure independent of the content of the events.

In case of an event bush **without cycles**, a "scenario" $\mathcal{SC}$ should be something like a "maximal part" of the corresponding structure graph $\mathcal{E}$ that does not include any alternatives:

- For any furcation chose exactly one of the incompatible alternatives.
- For any influx we have either influx, if external event is present, or accompanied flux, if external event is not present.

Alternative Scenarios - Example

We consider a structure graph similar to our ”transport example.

$$
\begin{tikzcd}
 & E_2 \arrow[d] & & & & \\
E_1 \arrow[r] \arrow[rr, bend left] & ifl_1 \arrow[d] & fl_1 \arrow[rrr] & & & E_4 \\
 & E_3 \arrow[r] & frc \arrow[r] \arrow[dr] & E_5 \arrow[r] & fl_2 \arrow[r] & E_6 \\
 & & & E_7 \arrow[r] & fl_3 \arrow[r] & E_8
\end{tikzcd}
$$

Scenarios in the example:

Dependent on the presence or non-presence of the primary external event E_2 and due to the furcation we should have here 3 alternative scenarios.

Three Sample Scenarios

$$\begin{tikzcd} E_1 \arrow[r] & fl_1 \arrow[r] & E_4 \end{tikzcd}$$

$$\begin{tikzcd} & E_2 \arrow[d] & & & & & \\ E_1 \arrow[r] & ifl_1 \arrow[r] & E_3 \arrow[r] & frc \arrow[r] & E_5 \arrow[r] & fl_2 \arrow[r] & E_6 \end{tikzcd}$$

$$\begin{tikzcd} & E_2 \arrow[d] & & & & & \\ E_1 \arrow[r] & ifl_1 \arrow[r] & E_3 \arrow[r] & frc \arrow[r] & E_7 \arrow[r] & fl_3 \arrow[r] & E_8 \end{tikzcd}$$

Attention:

Many open questions here! In principal, however, we should be able to work it out and extend it even to event bushes **with cycles**.

Directed Alternative Changes

What means "directed" for a scenario?

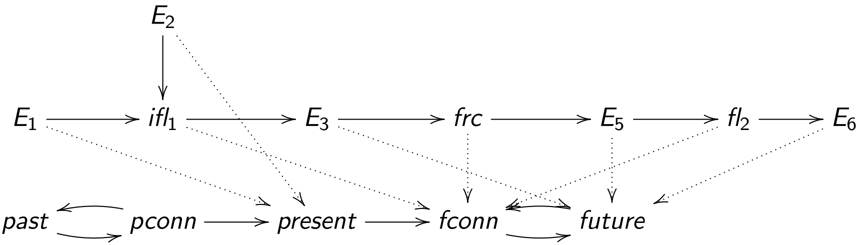
- There is **no cycle** in the scenario.
- At each "moment in time" the set of events is divided into present events, past events and future events, respectively.
- Correspondingly, the set of connectors is divided into connectors that have been "firing" once in the past and connectors that will "fire" in the future.
- The past influences the future only via the present events.
- All present events are independent from each other.

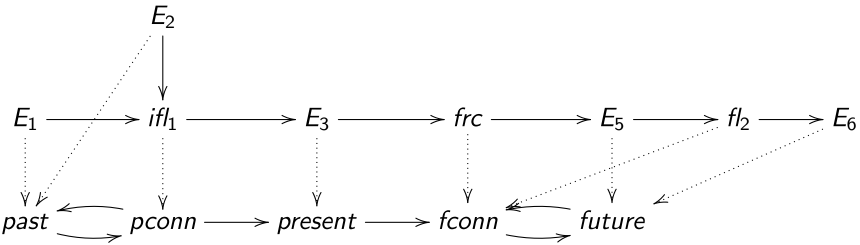
Snapshots (pictures)

We use the term **snapshot** for those "moments in time". And we formalize a **single snapshot** of a scenario $\mathcal{SC}$ as a graph homomorphism $\sigma : \mathcal{SC} \to \mathcal{P}$, where the (bipartite) graph $\mathcal{P}$ is given by

$$
\begin{tikzcd}
past \arrow[r, shift right] & pconn \arrow[l, shift right] \arrow[r] & present \arrow[r] & fconn \arrow[r, shift right] & future \arrow[l, shift right]
\end{tikzcd}
$$

Sample Snapshots 1





Sample Snapshots 2

$$
\begin{tikzcd}
 & E_2 \arrow[d] \arrow[ddl, dotted] & & & & & \\
E_1 \arrow[r] \arrow[d, dotted] & ifl_1 \arrow[r] \arrow[d, dotted] & E_3 \arrow[r] \arrow[dll, dotted] & frc \arrow[r] \arrow[dll, dotted] & E_5 \arrow[r] \arrow[dll, dotted] & fl_2 \arrow[r] \arrow[dll, dotted] & E_6 \arrow[dll, dotted] \\
past \arrow[r, bend right] & pconn \arrow[l, bend right] \arrow[r] & present \arrow[r] & fconn \arrow[r, bend right] & future \arrow[l, bend right] & &
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & E_2 \arrow[d] \arrow[ddl, dotted] & & & & & \\
E_1 \arrow[r] \arrow[d, dotted] & ifl_1 \arrow[r] \arrow[d, dotted] & E_3 \arrow[r] \arrow[dll, dotted] & frc \arrow[r] \arrow[dll, dotted] & E_5 \arrow[r] \arrow[dllll, dotted] & fl_2 \arrow[r] \arrow[dllll, dotted] & E_6 \arrow[dllll, dotted] \\
past \arrow[r, bend right] & pconn \arrow[l, bend right] \arrow[r] & present \arrow[r] & fconn \arrow[r, bend right] & future \arrow[l, bend right] & &
\end{tikzcd}
$$

Directed Alternative Changes

Observation

That a scenario $\mathcal{SC}$ is cycle free means, especially, that there exists for any event E in $\mathcal{SC}$ a snapshot $\sigma : \mathcal{SC} \to \mathcal{P}$ with $\sigma(E) = present$.

Moreover, directed means that any snapshot is uniquely determined by the "present" events!

What means now "directed changes"?

A scenario is progressing as a "cartoon", that is, as a sequence of **consecutive snapshots** where each event in the scenario has the status "present" in at least one of the snapshots of the "cartoon".

Directed Alternative Changes

Consecutive snapshots

Moving from one snapshot to a next one should be defined analogously to Petri nets:

- Given a snapshot certain connectors are "enabled".
- We chose an appropriate set of "enabled" connectors and "fire" them in parallel.
- This results in a new consecutive snapshot such that:
    - All the events, that are inputs of "fired" connectors, are moved from "present" to "past".
    - The "fired" connectors are moved from "future" to "past".
    - All the events, that are outcomes of "fired" connectors, are moved from "future" to "present.

Attention

There may be different ways to realize a scenario as a cartoon. All of them do have, however, the same start snapshot and the same final snapshot.

Thanks

Many thanks for your attention! Questions?

Exercises concerning graph homomorphisms

Exercise 1

Consider the following graphs

$$\mathcal{G}: \quad \begin{tikzcd} 1 & 3 \arrow[l] \arrow[r, bend right] & 4 \arrow[l, bend right] & 2 \arrow[l] \arrow[loop right] \end{tikzcd}$$

$$\mathcal{H}: \quad \begin{tikzcd} 1 & 2 \arrow[l] \arrow[r] & 3 & 4 \arrow[l] \end{tikzcd}$$

Find **all (!)** graph homomorphisms from $\mathcal{H}$ into $\mathcal{G}$. Explain why there is no graph homomorphisms from $\mathcal{G}$ into $\mathcal{H}$.

Exercise 2

Find two (finite) graphs $\mathcal{G}$ and $\mathcal{H}$ such that there exists neither a graph homomorphisms from $\mathcal{G}$ into $\mathcal{H}$ nor from $\mathcal{H}$ into $\mathcal{G}$. Argue why there are no graph homomophisms. (**Hint:** It's quite enough to look for graphs with two or three vertices, respectively.)

Composition of graph homomorpisms

Given two graph homomorphisms $\phi : \mathcal{G} \to \mathcal{H}$ and $\psi : \mathcal{H} \to \mathcal{K}$ the composition $\phi_V ; \psi_V : G_V \to K_V$ of the underlying functions $\phi_V : G_V \to H_V$ and $\psi_V : H_V \to K_V$ provides a graph homomorphism $\phi; \psi : \mathcal{G} \to \mathcal{K}$: $(x, y) \in G_E$ entails $(\phi_V(x), \phi_V(y)) \in H_E$ and this entails, in turn, $(\psi_V(\phi_V(x)), \psi(\phi_V(y))) = (\phi_V ; \psi_V(x), \phi_V ; \psi_V(y)) \in K_E$.

Type graphes and typed graphs

If we have a graph homomorphism $\tau_{\mathcal{G}} : \mathcal{G} \to \mathcal{T}$ we can consider $\mathcal{T}$ as a "type graph", i.e., as a graph with "types" as vertexes, thus $\mathcal{G}$ becomes a "graph typed over $\mathcal{T}$" (via $\tau_{\mathcal{G}}$).